Investigation of Laminar Flow in a Porous Pipe with Variable Wall Suction

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An analytical model has been developed to provide an analogy for and predict flow property distributions in the condensing regions of a heat pipe. The model incorporates the phenomenon of incompressible, steady, laminar flow through a uniformly-porous-wall pipe with a closed end-wall. A Kármán-Pohlhausen momentum integral technique was employed to reduce the axisymmetric Navier-Stokes equation to a nonlinear second-order ordinary differential equation. The axial pressure gradient, suction velocity at the porous wall, and wall friction coefficient were computed over a range of Reynolds numbers and wall permeabilities. These flow property distributions were not constrained a priori, but were allowed to vary in accordance with the conservation laws. The calculated flow property distributions for the condenser region of the heat pipe model are compared with published experimental data and several other theoretical models cited in the literature.

Nomenclature

c_f = friction coefficient
$f(z)$ = velocity function
k = wall permeability in Darcy's Law
k_1 = parameter, $k/\mu t$
L = length of porous tube
N_R = axial inlet flow Reynolds number, $\bar{w}_o R/\nu$
Re_r = radial ejection or suction Reynolds number, $\bar{u}_w R/\nu$
$\bar{p}$ = axial static pressure
p = nondimensional axial static pressure, $\bar{p}/\rho \bar{w}_o{}^2$
$\bar{r}$ = radial coordinate
r = nondimensional radial coordinate
R = mean radius of porous tube
t = thickness of porous wall
$\bar{u}$ = radial velocity
u = nondimensional radial velocity, $\bar{u}/\bar{w}_o$
$\bar{w}$ = axial velocity
w = nondimensional axial velocity, $\bar{w}/\bar{w}_o$
x = axial coordinate
z = nondimensional axial coordinate
Λ = flow parameter, Rt/k
μ = fluid viscosity
ν = kinematic viscosity
ρ = fluid density

Subscripts

w = value at porous wall
e = conditions outside porous pipe
i = conditions inside porous pipe
o = conditions at porous pipe inlet

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Introduction

A NUMBER of models have been developed to describe the flow properties in porous tubes, and several have been extended for application to heat pipe performance. In these studies of internal heat pipe conditions, the similitude between vapor flow through a cylindrical porous wall pipe with suction to the condenser region has been suggested as a means of determining the vapor pressure variations. Analytical studies of the axial pressure variations and velocity profiles in laminar flow have been reported, usually with the restrictive assumption of uniform suction at the wall. A relatively small amount of experimental data has been presented for the uniform suction boundary condition. Although the objective of such investigations has been to develop an insight into the variable internal pipe flow properties, the boundary condition of uniform suction at the wall is usually inconsistent with this objective. The purpose of this paper, then, is to present an analytical model for the representation of the flow properties in the condenser section of a heat pipe. The analysis is based on laminar, incompressible, steady flow in a uniformly-porous-wall pipe with variable wall suction.

One of the early formulations of the governing equations of an analytical model for performance of a heat pipe was developed by Cotter.[1] Modifications to this theory have been advanced by many investigators for the liquid-vapor processes taking place within various heat pipe configurations. Alternative models for the fluid dynamical pressure drop have been developed to evaluate the vapor-liquid univiscous interaction and condensation.[2–5] In most cases, comparisons with experiments have not been presented because of the difficulties of precision measurement of property distributions within the liquid-vapor regions. Analytical developments have been extended to include the effects of noncondensable gases at the condenser end of the heat pipe. The application of that concept was advanced by Grover et al.[6] and Katzoff[7] as a technique for thermal control by providing a variable thermal conductor heat pipe with inert gases and vapor.[8,9] Again, almost no experimental data is available for comparison between predicted and measured property and species distributions.

A basic assumption in the majority of models is that the flow of vapors in the evaporator and condenser is dynamically equivalent to pipe flow with injection or suction at the wall to simulate evaporation and condensation. Extension of such models from open-end channel flow or pipe flow following Yuan and Finkelstein[10] or Eckert et al.[11] is made to justify the assumed velocity profile distributions and pressure field. The adequacy of this hypothesis is questionable since the appropriate comparison must be made using a closed end-wall tube with an impermeable surface. In the analysis of Galowin and Barker[12] for the closed planar heat pipe, the velocity vanished at both the evaporator and condenser end walls. Somogyi and Yen[9] satisfied the condenser end condition with the assumption of zero velocity.

Most of the analyses which have treated viscous flows through open-end porous tubes and channels have also been restricted by the a priori specification of the normal wall velocity and/or wall shear stress. The evaporation-condensation conditions at the wall are variable as a result of the nonuniform heat input or rejection, or the axial conduction. The wall condition, therefore, must be established from the governing equations rather than arbitrarily prescribed. Such analyses were included in Refs. 9 and 12 where the variable normal wall velocity was recognized as the forcing function for the system.

In the experimental study by Quaile and Levy,[13] the simulation of the vapor flow in the condenser region by fluid suction at the wall of a porous tube was investigated. The experimental results were compared with analytical "similarity" solutions and "inlet" solutions, and significant departure was observed throughout the laminar flow regime. For example, the axial static pressure distribution along the tube length differed by as much as a factor of 4 in some cases.

The present analysis provides the axial pressure variation and end-wall pressures within a porous wall cylinder with an impermeable end-wall as an analog of the condenser section of a heat pipe. The results are restricted to the vapor space of a heat pipe and are completely decoupled from the wick and liquid at the walls. The results are compared with published experimental data of Quaile and Levy[13] and other theoretical models cited in the literature.

Method of Analysis

The theoretical analysis of laminar pipe flow in a porous wall cylinder utilized by Galowin and DeSantis[14] to study the effects of variable wall injection and suction in a porous tube has been adopted for the present analysis. Their method represents the fluid dynamical portion of an investigation to develop a technique for predicting the condensation-front location in the condenser section of a heat pipe. The condensation-front location will be accounted for by considering the extent of the super-cooled vapor state as axial pressure variations occur, i.e., the rate of change of saturation temperature with pressure. The present analysis considers the development of pipe flow within a uniformly porous wall tube. The axially varying suction velocity at the wall is created by the local pressure difference between the internal and external pressures. Since the pressure difference does not remain constant, a variable normal wall velocity distribution will exist along the length of the tube.

The Kármán-Pohlhausen integral momentum technique is adopted[14,15] and fully developed Hagen-Poiseuille pipe flow is postulated at the inlet. For the large length-to-diameter ratios treated herein, the mean static pressure is assumed to be only a function of the axial coordinate, and the axial flow Reynolds number is assumed to be much greater than one. A quadratic profile is chosen to simplify the integrodifferential equation and thereby obtain a nonlinear ordinary differential equation. The magnitude of the normal (suction) velocity at the wall is small and varies with the axial coordinate only. The flow is axisymmetric, and Darcy's Law is assumed to describe the flow through the porous wall. Numerical solutions for velocity, axial static pressure, and friction coefficient are obtained for specified geometries and flow parameters.

The choice of the Hagen-Poiseuille flow at the inlet to simulate a condenser is justified since the adiabatic section of a heat pipe acts as a solid wall tube (neglecting miniscus liquid-vapor interaction) which provides fully developed pipe flow at the inlet to the porous section. The length-to-diameter ratio of the porous tube is much greater than one, thus the approximations for the radial velocity and static pressure are valid. In addition, it is implied in the analysis (through Darcy's Law) that the axial velocity is much larger than the radial velocity.

The simplified momentum and continuity equations in nondimensional form are

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{dp}{dz} + \frac{1}{N_R}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right] \tag{1}$$

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \tag{2}$$

with boundary conditions:

$$\begin{aligned} r = 0; &\quad \partial w/\partial r = 0 \\ r = 1; &\quad w = 0, \quad u = u_w(z) \\ z = 0; &\quad w = (1 - r^2) \end{aligned} \tag{3}$$

The integral form of the momentum equation is obtained by integrating Eq. (1) with respect to a mean radius r between the limits $r = 0$ and $r = 1$, and using the continuity equation (2) to yield

$$\frac{d}{dz}\int_0^1 rw^2\,dr = -\frac{1}{2}\frac{dp}{dz} + \frac{1}{N_R}\left[\left(\frac{\partial w}{\partial r}\right)_w + \int_0^1 r\frac{\partial^2 w}{\partial z^2}\,dr\right] \tag{4}$$

The wall thickness of the porous tube is taken into account by utilizing a mean diameter rather than the i.d. of the tube. A quadratic profile for w is assumed in the form

$$w = \bar{w}/\bar{w}_o = [1 + f(z)](1 - r^2) \tag{5}$$

This equation satisfies the boundary conditions and leads to a relatively simple ordinary differential equation. More exact forms with variable coefficients for the nonsimilar flow require extensive numerical methods of solution.[12] The function $f(z)$ represents the change in the axial velocity from the assumed Hagen-Poiseuille inlet profile as the flow moves through the porous tube. The functional relation of Eq. (5) satisfies the appropriate flow conditions, i.e., the axial velocity vanishes at the wall; the velocity profile is symmetrical at the centerline; it is fully developed at the inlet where $f(z) = 0$ at $z = 0$; and it vanishes at the end wall for $f(z) = -1$ at $z = 1$. The normal wall velocity, $u_w(z)$, is obtained by integration of the continuity equation (2) with the assumed axial velocity profile, Eq. (5), to yield

$$u_w = \frac{\bar{u}_w}{\bar{w}_o} = -\frac{1}{4}\frac{df}{dz}; \quad \frac{\bar{u}_w}{\bar{w}} = -\frac{3}{8(1+f)}\frac{df}{dz} \tag{6}$$

where u_w vanishes at the centerline. The flow through the porous wall is governed by Darcy's Law

$$-\partial\bar{p}/\partial\bar{r} = (\mu/k)\bar{u}_w \tag{7}$$

which can be written in the simplified form

$$\bar{u}_w(z) = -(k/\mu)(\bar{p}_e - \bar{p}_i)/t \tag{8}$$

for the porous cylinder. The pressure difference between the exterior and interior of the pipe represents a reasonable approximation to the pressure gradient across the pores. For the special case of outflow from within the porous tube, the internal pressure must exceed the external pressure, and the difference may be considered simply the gage pressure and will be designated as the pressure $\bar{p}$ within the pipe. Consequently, Eq. (8) can be expressed as

$$\bar{u}_w(z) = k_1\bar{p} \tag{9}$$

Comparing with Eq. (6) and casting in nondimensional terms, Eq. (9) can be written as

$$p(z) = -(\Lambda/4N_R)\,df/dz \tag{10}$$

Thus the pressure within the pipe is related to the velocity function f and the pressure gradient term of Eq. (4) can be replaced by Eq. (10).

Upon substitution of Eqs. (5) and (10) into Eq. (4), the resulting nonlinear ordinary differential equation containing only the velocity function f and its derivatives is obtained in the form

$$\frac{1}{4N_R}\left(1+\frac{\Lambda}{2}\right)\frac{d^2f}{dz^2}-\frac{1}{3}(1+f)\frac{df}{dz}-\frac{2}{N_R}(1+f)=0 \quad (11)$$

and the boundary conditions are

$$z=0;\quad f=0$$
$$z=1;\quad f=-1 \quad (12)$$

The friction coefficient is obtained from the shear stress at the wall, which is in nondimensional terms

$$c_f=-\frac{1}{N_R}\left[4(1+f)+\frac{1}{2}\frac{d^2f}{dz^2}\right] \quad (13)$$

The quantity d^2f/dz^2 is proportional to the rate of change of the axial static pressure along the porous pipe. It is small in comparison to the first term along the tube length and can be neglected. Thus the simplified form for the friction coefficient is

$$c_f \cong \frac{4}{N_R}(1+f) \quad (14)$$

In addition to the axial inlet Reynolds number, a Reynolds number based on the radial ejection through the porous wall is defined as

$$Re_r = \bar{u}_w R/\nu = -(N_R/4)\,df/dz \quad (15)$$

Finally, from Eqs. (6) and (10), the wall velocity can be obtained in terms of the static pressure, i.e.,

$$u_w(z) = (N_R/\Lambda)p(z) \quad (16)$$

Therefore, the analytical model is defined completely in terms of the Reynolds numbers, a velocity function, f (which is fundamentally the axial centerline velocity), and a geometric-flow parameter, Λ. The flow property distributions, $p(z)$, $c_f(z)$, and $u_w(z)$ are then computed directly from Eqs. (10, 14, and 16), respectively.

Results and Discussion

A fourth-order Runge-Kutta technique was used to integrate the second-order, ordinary nonlinear differential equation subject to the condition that the axial velocity vanishes at the end wall of the porous tube section. The parameters which were varied are the axial Reynolds number and the flow parameter Λ which is related to the wall permeability. The quantity Λ is computed directly from a knowledge of the porous tube geometry, the total flow rate, and the pressure drop through the porous wall. The numerical marching procedure is terminated when the centerline axial velocity vanishes, i.e., when $f = -1$. At that point in the integration, the end wall of the porous tube has been reached and the problem is completed.

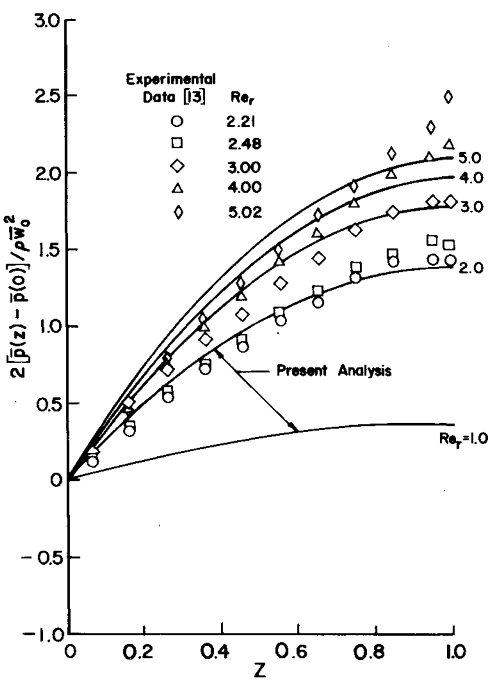

Fig. 1 Variation of the axial static pressure in a porous wall tube—comparison with experimental data.[13]

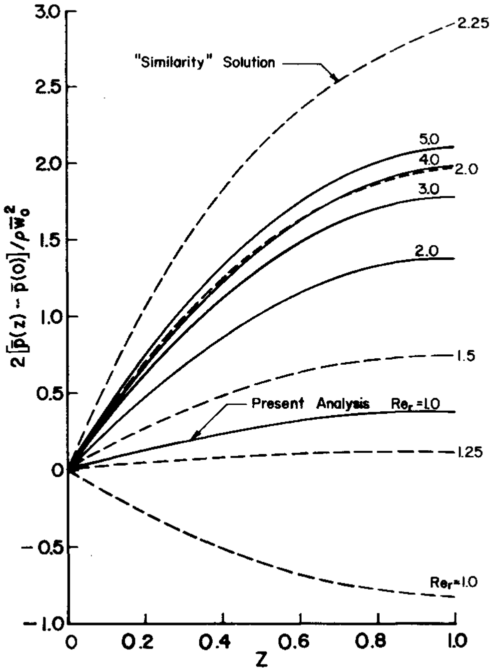

Fig. 2 Variation of the axial static pressure in a porous wall tube—comparison with "Similarity" solutions.[13]

Detailed comparisons of the behavior of the axial static pressure and radial Reynolds number were made between the present analytical model and the experiments of Quaile and Levy.[13] The variation of the axial static pressure along the porous tube for several radial Reynolds numbers between 1.0 and 5.0, and for $\Lambda = 2.5 \times 10^6$, is shown in Fig. 1. The theoretical curves of the present analysis are depicted by the solid lines. For purposes of comparison, the radius associated with the experimental data was taken to be the inside radius of the tube, while in the analytical solution, a mean radius which includes the effect of wall thickness was employed. By modifying the analytical results to correspond to the inside radius, the curves of Fig. 1 are obtained.

Excellent agreement between theory and experiment is observed for radial Reynolds number of 2–5. For a radial Reynolds number of 5, the experimental data appear to diverge from the theoretically predicted value at the end of the tube. The agreement with the experimental data, however, is far better than the "similarity" solutions which is emphasized by the comparison shown in Fig. 2. The axial static pressure obtained in the experimental investigation varies by approximately a factor of 2 at the end wall of the porous tube. This magnitude is in close agreement with the results obtained in the present analysis. The "similarity" solutions shown in Fig. 2, however, indicate an end wall static pressure variation of nearly one order of magnitude in the radial Reynolds number range of 1.5–5.0.

Figure 3 depicts the over-all axial pressure drop in the porous tube as a function of radial Reynolds number for the experimental data, "similarity" and "inlet region" solutions,[13] and the present analysis. Excellent agreement is obtained over $1 < Re_r < 5$ between the laboratory measurements and the new model. The composite of several "similarity" solutions shows

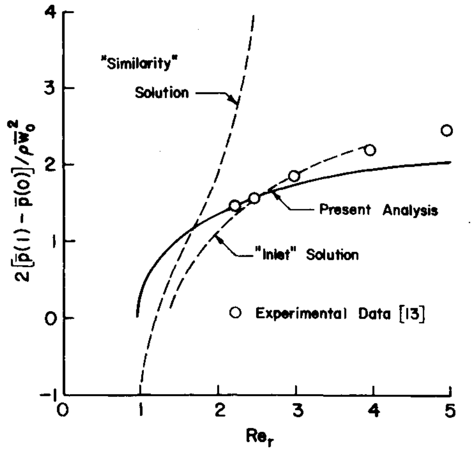

Fig. 3 Variation of the axial static pressure with radial Reynolds number —comparison with "Inlet" and "Similarity" solutions.[13]

little or no agreement with experimental data, and the "inlet" solutions correctly predict the over-all pressure drop only over a limited range of radial Reynolds numbers. The primary reasons that the present analysis correctly predicts the flow property distributions are 1) the suction velocity and friction coefficient are permitted to vary along the wall of the tube; and 2) the permeability of the tube is a parameter which can be varied to correspond to specific experimental conditions such as pipe material and porosity.

The variation of the wall suction velocity as a function of the tube length is given in Fig. 4. The curves emphasize the physically realistic condition of a variable pressure drop across the porous tube along its length. The results are shown for $2 \leq Re_r \leq 5$ and $\Lambda = 2.5 \times 10^6$. The curves are dashed as the end of the tube is approached, since the axial velocity approaches zero (and is zero) at the nonpermeable end-wall of the tube. Thus the velocity ratio becomes infinite in the mathematical sense since the model does not take into account the discontinuity at the corner of the end-wall.

The Re_r and Λ values used in Fig. 4 are the same as those used in Fig. 1, in which excellent agreement is obtained between the experimental data and the present model. Therefore, the intrinsic construction of the present analysis is sufficiently justified, i.e., the suction velocity and wall shear stress are allowed to vary according to the conservation laws.

The present analysis was extended to determine its versatility for prediction of flow distributions for turbulent porous pipe flow in the axial Reynolds number range below 10,000. The purpose was to determine if estimates of the wall friction coefficient could be obtained for turbulent flow with a laminar flow analysis. To achieve this objective, a limited comparison was made with the theoretical analysis of Kinney and Sparrow[16] for turbulent flow in a tube with surface suction. Figure 5 compares the friction factor variation with suction velocity for the turbulent analysis and the present model. The inlet axial Reynolds number was chosen to be 10,000 to correspond to the turbulent flow analysis. Excellent agreement between the two models was obtained, as seen in Fig. 5, over the range of wall suction velocities considered. Therefore, the present model can be used to estimate the friction coefficient for low Reynolds number turbulent flow in a porous tube. However, the laminar flow analysis described herein should be confined only to estimates of this type, and the more sophisticated turbulent flow analysis should be used in general.

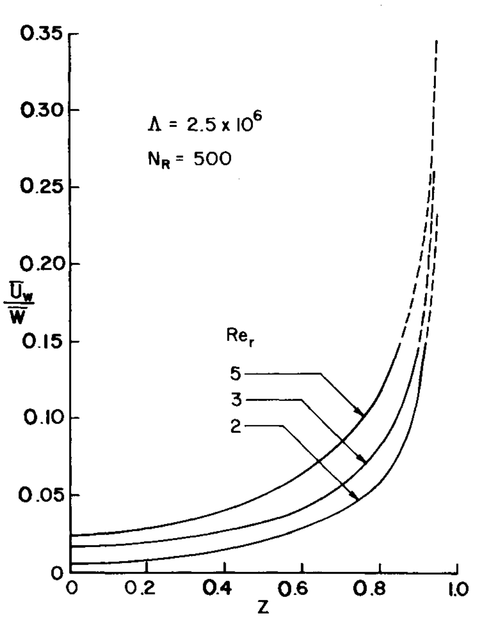

Fig. 4 Variation of the radial suction velocity with tube length.

Summary and Conclusions

A new model for the prediction of flow property distributions in a porous tube has been developed. The mathematical model described herein leads to a simple nonlinear ordinary differential equation which can be integrated numerically. More exact forms with variable coefficients require extensive numerical methods as described in Ref. 12.

The present theoretical analysis shows excellent agreement with available experimental data over a specified radial Reynolds number range in the laminar flow regime. The excellent agreement between theory and experiment results from the versatility of the model, namely 1) the suction velocity and wall friction coefficient are allowed to vary according to conservation principles, and 2) the wall permeability is introduced as a parameter to more realistically represent the wall material and porosity. Furthermore, the present analysis provides more accurate predictions of flow properties in a porous tube than the previously developed "similarity" solutions. The dynamical similarity between the present porous tube flow analysis and the vapor flow in the condenser region of a heat pipe will provide estimates of heat pipe performance.

The model can be extended in a limited manner to obtain estimates of the friction coefficient in a turbulent pipe flow with suction at the porous walls. Good agreement with a turbulent flow analysis was obtained for a flow Reynolds number of less than 10,000. The present analysis may also be modified by incorporating pressure distributions which vary with both the

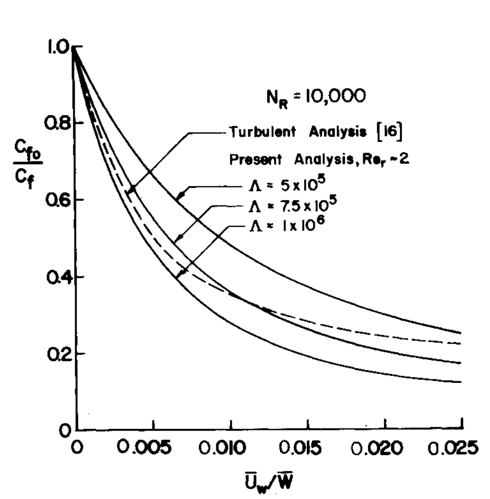

Fig. 5 Variation of the local friction factor with radial suction velocity—comparison with the turbulent flow analysis.[16]

axial and radial coordinates, and by prescribing more complicated inlet velocity profiles to represent specific heat pipe applications.

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